International Journal of Mathematics and Physical Sciences Research ISSN 2348-5736 (Online)
Vol. 6, Issue 1, pp: (61-71), Month: April - September 2018, Available at: www.researchpublish.com

THE SQAURE ROOT OF A 2X2 PERFECT SQUARE MATRIX

Emmanuel Nwadim

Department of Electrical Engineering, Nnamdi Azikiwe University, Awka, Nigeria.

Abstract: **A perfect square is a number that can be expressed as the product of two equal integer, hence the square root of a perfect square gives two equal integers.**

In this study, we introduce a new method for finding the square root of a 2x2 matrix using one of the property of determinant that states that if a matrix C is obtained from a matrix A by multiplying it with matrix B of the same order, the determinant of C is the product of the determinants of A and B.

That is if we have a matrix C such that A.B = C, then the above statement can be written as;

$$\Delta_C = \Delta_A \, . \, \Delta_B$$

But in this case $\Delta_A = \Delta_B$, so $\Delta_C = \Delta_A{}^2$.

Keywords: **square root, perfect square, method.**

1. INTRODUCTION

If we change the problem into system of equations, then we get an inconsistent system of solutions.

The breakthrough to this was achieved by a ratio I called the "**UNCERTAINTY RATIO**" and is backed up by some principles.

The following theorem solves our problem

2. THEOREM 1

THE DETERMINANT METHOD (PROPERTY)

It states that the determinant of a squared matrix is the square of the determinant of its original matrix.

i.e. **A.A = B**

$\Delta_B = \Delta_A{}^2$

Where Δ_B is the determinant of the squared matrix

Δ_A is the determinant of the original matrix

Proof:

Let **A** be a 2 x 2 matrix containing all elements as integers be square to give a matrix **B**.

A = $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ **B** = $\begin{pmatrix} e & f \\ g & h \end{pmatrix}$

A.A =B

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

$$\begin{pmatrix} a^2 + bc & ab + bd \\ ac + cd & d^2 + bc \end{pmatrix} = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

The determinant is, $\begin{vmatrix} a^2+bc & ab+bd \\ ac+cd & d^2+bc \end{vmatrix}$

$\Delta_2 = (a^2 + bc)(bc + d^2) - (ac + cd)(ab + bd)$

Expanding

$\Delta_2 = a^2bc + a^2d^2 + b^2c^2 + bcd^2 - (a^2bc + abcd + abcd + bcd^2)$

$\Delta_2 = a^2bc + a^2d^2 + b^2c^2 + bcd^2 - a^2bc - 2abcd - bcd^2$

$\Delta_2 = a^2d^2 - 2abcd + b^2c^2$

Factorizing

$\Delta_2 = a^2d^2 - abcd - abcd + b^2c^2$

$\Delta_2 = ad(ad-bc) - bc(ad-bc)$

$\Delta_2 = (ad-bc)(ad-bc)$

It can be observed that the determinant of the original matrix $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ is (ad – bc)

$\Delta_1 = (ad - bc)$

$\therefore \Delta_2 = \Delta_1^2$

Where Δ_2 is the determinant of the of the squared matrix

Δ_1 is the determinant of the original matrix

3. NUMERICAL EXAMPLES

1. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}^2 = \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix}$

$\Delta_1 = 4(1) - 3(2)$

$\Delta_1 = 4 - 6$

$\Delta_1 = -2$

$\Delta_2 = 22(7) - 15(10)$

$\Delta_2 = 154-150$

$\Delta_2 = 4$

$\therefore \Delta_2 = \Delta_1^2$

2. $\begin{pmatrix} 1 & 4 \\ 3 & -2 \end{pmatrix}^2 = \begin{pmatrix} 13 & -4 \\ -3 & 16 \end{pmatrix}$

$\Delta_1 = -2(1) - 4(3)$

$\Delta_1 = -2 - 12$

$\Delta_1 = -14$

$\Delta_2 = (16x13) - (-4 \text{ x } -3)$

$\Delta_2 = 208 - 12$

$\Delta_2 = 198$

$\therefore \Delta_2 = \Delta_1^2$

4. THEOREM 2

Determining The Square Root Of A 2 X 2 Perfect Square Matrix

Proof:

Let **A** be a 2 x 2 matrix of $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and it is squared to give $\begin{pmatrix} e & f \\ g & h \end{pmatrix}$ where **a, b, c, d, e, f, g, h, €Z**

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^2 = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

$$\begin{pmatrix} a^2 + bc & ab + bd \\ ac + cd & d^2 + bc \end{pmatrix} = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

$a^2 + bc = e$ ………….(i)

$ab + bd = f$ ………….(ii)

$ac + cd = g$……….....(iii)

$bc + d^2 = h$ ………….(iv)

From eqn (ii),

$b(a+d) = f$…………….(ii)

From eqn (iii),

$c\,(a + d) = g$ …………(iii)

Dividing eqn (iii) by (ii)

$$\frac{c(a+d)}{b(a+d)} = \frac{g}{f}$$

$$\frac{c(a+d)}{b(a+d)} = \frac{g}{f}$$

$$\frac{c}{b} = \frac{g}{f} \qquad \{c \ \ \& \ \ b \textit{ are gotten from the } \text{factors of the ratio g : f}\}$$

This ratio is called the "**UNCERTAINTY RATIO**"

Equation (i) and (iv) are used to test the correct factors of "b" and "c" obtained in the uncertainty ratio (i.e. both equation gives a perfect square number).

5. EXTENSION (PRINCIPLES OF THE UNCERTAINTY RATIO)

1. If "**g**" and "**f**" has the negative sign, it should not be cancelled in the uncertainty ratio.
2. If "**g**" has the negative sign, it should not be transferred to "**f**" because the shows that the integer "c" actually carry a negative sign.
3. If "**f**" has the negative sign, it should not be transferred to "**g**" because it shows that the integer "**b**" actually have minus sign.

6. SOLVING FOR "A" AND "D"

Equation (i) and (iv) are used to obtain integer values of "**a**" and "**d**" respectively.

Extension

The check the polarity of "**a**" and "**d**"

$a = \pm\alpha$

$d = \pm ɗ$

:. The original matrix becomes $\begin{pmatrix} \pm\alpha & b \\ c & \pm ɗ \end{pmatrix}$

and the squared matrix is $\begin{pmatrix} e & f \\ g & h \end{pmatrix}$

Recall

$\Delta_2 \quad = \quad \Delta_1^{\,2}$

$\Delta_1 \quad = \quad \pm\sqrt{\Delta_2}$

Where Δ_1 is the determinant of the original matrix

Δ_2 is the determinant of the squared matrix

Therefore, the polarity of "**a**" or "**d**" depends on the determinant, the numerical examples below gives a better understanding of this statement.

NOTE (Principles of the uncertainty ratio)

4 If there is to be negative sign between "**a**" or "**d**" , it goes to the value with the smaller number by comparing "**e**" and "**h**"

The squared matrix is $\begin{pmatrix} e & f \\ g & h \end{pmatrix}$

"**a**" takes minus if **e < h**

"**d**" takes minus if **h < e**

When **e = h**, '**a**" and "**d**" are taken as positive and a case where "**ad**" is positive, both "**a**" and "**d**" are taken as positive and **not** negative.

7. NUMERICAL EXAMPLES

1. Find the square root of $\begin{pmatrix} 40 & -3 \\ -12 & 13 \end{pmatrix}$

Solution

. $\begin{pmatrix} e & f \\ g & h \end{pmatrix}^{\frac{1}{2}}$ = . $\begin{pmatrix} 40 & -3 \\ -12 & 13 \end{pmatrix}^{\frac{1}{2}}$

Recall,

$$\frac{c}{b} = \frac{g}{f} = \frac{-12}{-3}$$

Possible values

c	**b**
-12	-3
-4	-1

There are two possible values for "**b**" and "**c**", so we test them using eqn **(i)** and **(iv)**

a^2 + bc = e ……….. (ii) **bc + d^2 = h ………. (iv)**

When **c** = -12, **b** = -3

$a^2 + (-12x-3) = 40$	$(-12x-3) + d^2 = 13$
$a^2 + 36 = 40$	$36 + d^2 = 13$
$a^2 = 40 - 36$	$d^2 = 13 - 36$
$a^2 = 4$	$d^2 = -23$
$a = \pm 2$	$d = \sqrt{-23}$

a= ±2 & **d** = √-23 (this shows that -12 and -3 are not the correct factors)

When **c** = -4, **b** = -1

$a^2 + bc = e$	$bc + d^2 = h$
$a^2 + (-4 \times -1) = 40$	$(-4 \times -1) + d^2 = 13$
$a^2 + 4 = 40$	$4 + d^2 = 13$
$a^2 = 40 - 4$	$d^2 = 13 - 4$
$a^2 = 36$	$d^2 = 9$
$a = \pm 6$	$d = \pm 3$

The original matrix would be of the form $\begin{pmatrix} \pm 6 & -1 \\ -4 & \pm 3 \end{pmatrix}$

TO CHECK THE polarity of variable 6 and 3

The determinant of the squared matrix Δ_2 is

$$\begin{vmatrix} 40 & -3 \\ -12 & 13 \end{vmatrix}$$

Δ_2 = (40 x 13) – (-12 x -3)

Δ_2 = 520 – 36

Δ_2 = 484

Recall

$\Delta_1 = \pm\sqrt{\Delta_2}$

$\Delta_1 = \pm\sqrt{484}$

$\Delta_1 = \pm 22$

The determinant of the original matrix of the form $\begin{bmatrix} \pm 6 & -1 \\ -4 & \pm 3 \end{bmatrix}$ is

Δ_1 = (±6 x ±3) – (-1 x -4)

Δ_1= ±18-4

When the variable 18 is positive, it gives 14 ≠ -22

When the variable 18 is negative, it gives – 22 = -22

So either "**a**" or "**d**" takes the negative sign and since **h** < **e**

i.e (13 < 40), "**d**" takes minus

:. The original matrix becomes $\begin{pmatrix} 6 & -1 \\ -4 & -3 \end{pmatrix}$ or its negative $\begin{pmatrix} -6 & 1 \\ 4 & 3 \end{pmatrix}$

(2) Find the square root of the matrix $\begin{pmatrix} 7 & -6 \\ -9 & 22 \end{pmatrix}$

Solution

Recall,

$$\frac{c}{b} = \frac{g}{f} = \frac{-9}{-6} = \frac{-3}{-2}$$

Possible values

c	**b**
-9	-6
-3	-2

There are two possible values for "**b**" and "**c**", so we test them using eqn **(i)** and **(iv)**

a^2 + bc = e ….. (i) **bc + d^2 = h . …… (iv)**

When **c** = -9, **b** = -6

a^2+ (-9 x -6) = 7 (-9 x -6) + **d^2** = 22

a^2 + 54 = 7 54 + **d^2** = 22

a^2 = -47 **d^2** = -32

:. **b** ≠ -6 and **c** ≠ -9

To test for **c** = -3 and **b** = -2

$\mathbf{a}^2 + (-3 \times -2) = 7$	$(-3 \times -2) + \mathbf{d}^2 = 22$
$\mathbf{a}^2 + (6) = 7$	$(6) + \mathbf{d}^2 = 22$
$\mathbf{a}^2 = 7 - 6$	$\mathbf{d}^2 = 22 - 6$
$\mathbf{a}^2 = 1$	$\mathbf{d}^2 = 16$
$\mathbf{a}^2 = \pm\sqrt{1}$	$\mathbf{d}^2 = \pm\sqrt{16}$
$\mathbf{a} = \pm 1$	$\mathbf{d} = \pm 4$

The original matrix would be of the form $\begin{pmatrix} \pm 1 & -2 \\ -3 & \pm 4 \end{pmatrix}$

To check the polarity of "**a**" and '**d**"

The determinant of the squared matrix Δ_2 is $\begin{vmatrix} 7 & -6 \\ -9 & 22 \end{vmatrix}$

$\Delta_2 = 22(7) - (-9 \times -6)$

$\Delta_2 = 154-54$

$\Delta_2 = 100$

Recall,

$\Delta_1 = \pm\sqrt{\Delta_2}$

$\Delta_1 = \pm\sqrt{100}$

$\Delta_1 = \pm 10$

Where, Δ_1 is the determinant of the original matrix.

The determinant of the original matrix Δ_1 is $\begin{bmatrix} \pm 1 & -2 \\ -3 & \pm 4 \end{bmatrix}$

$\Delta_1 = (\pm 1 \times \pm 4) - (-3 \times -2)$

$\Delta_1 = \pm 4 - 6$

When the variable 4 is positive, it gives $-2 \neq \pm 10$

When the variable 4 is negative, it gives -10 = -10

So either "**a**" or "**d**" takes minus

And since e < h, "**a**" takes the negative sign

:. The original matrix becomes $\begin{pmatrix} -1 & -2 \\ -3 & 4 \end{pmatrix}$ or its negative $\begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}$

8. SPECIAL CASES (LIMITATIONS)

1. In this case, the determinant method does not apply here and it occurs when either "b" or "c" is zero

$$\begin{pmatrix} a & 0 \\ c & d \end{pmatrix}\begin{pmatrix} a & 0 \\ c & d \end{pmatrix} = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

$$\begin{pmatrix} a^2 & 0 \\ ac + cd & d^2 \end{pmatrix} = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

$\mathbf{a}^2 = \mathbf{e}$ **…. (i):.** $\mathbf{a} = \pm\sqrt{\mathbf{e}}$

$\mathbf{ac + cd = g}$ **…. (ii)**

$\mathbf{d}^2 = \mathbf{h}$ **…(iii),** $\mathbf{d} = \pm\sqrt{\mathbf{h}}$

From eqn**(ii)**

c(a+d) = g

$$c = \frac{g}{(a+d)}$$

Therefore the value of **(a + d)** should be a factor of **g** since **a, c, d, e, f, g, €Z̰**

The same applies **when c = 0 (when c=0, g=0)**

In such cases, the square root gives four possible square matrices depending on the factors obtained as "**c**"

9. NUMERICAL EXAMPLES

(i) Find the square root of $\begin{pmatrix} 1 & 0 \\ 8 & 9 \end{pmatrix}$

Solution

Automatically **b** = 0 since **f** = 0

Recall, $\mathbf{a}^2 = \mathbf{e}$ $\qquad \mathbf{d}^2 = \mathbf{h}$

$\mathbf{a}^2 = 1$ $\qquad \mathbf{d}^2 = 9$

$\mathbf{a} = \pm 1$ $\qquad \mathbf{d} = \pm 3$

$$c = \frac{g}{(a+d)}$$

When **a** = +1 and c = +3

$$c = \frac{8}{(1+3)} = \frac{8}{4} = 2 \text{ the matrix becomes } \begin{pmatrix} 1 & 0 \\ 2 & 3 \end{pmatrix} \text{ or } \begin{pmatrix} -1 & 0 \\ -2 & -3 \end{pmatrix}$$

When **a** = -1, and **c** = 3

$$c = \frac{8}{(-1+3)} = \frac{8}{2} = 4 \text{ the matrix becomes } \begin{pmatrix} -1 & 0 \\ 4 & 3 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & 0 \\ -4 & -3 \end{pmatrix}$$

Hence, the square root of $\begin{pmatrix} 1 & 0 \\ 8 & 9 \end{pmatrix}$ gives four possible matrices

2. Find the square root of $\begin{pmatrix} 16 & -14 \\ 0 & 9 \end{pmatrix}$

Solution

Automatically **c=0,** since **g = 0**

Recall, $\mathbf{a}^2 = \mathbf{e}$ $\qquad \mathbf{d}^2 = \mathbf{h}$

$\mathbf{a}^2 = 16$ $\qquad \mathbf{d}^2 = 9$

$\mathbf{a} = \pm 4$ $\qquad \mathbf{d} = \pm 3$

$$b = \frac{f}{(a+d)}$$

When **a** = +4 and **d** = +3

$$b = \frac{-14}{(4+3)} = \frac{-14}{7} = -2, \text{ The matrix becomes } \begin{pmatrix} 4 & -2 \\ 0 & 3 \end{pmatrix} \text{ or } \begin{pmatrix} -4 & 2 \\ 0 & -3 \end{pmatrix}$$

When **a** = -4 and **d** = +3

b $= \frac{-14}{(-4+3)} = \frac{-14}{-1} = 14$, The matrix $\begin{pmatrix} -4 & 14 \\ 0 & 3 \end{pmatrix}$ or $\begin{pmatrix} 4 & -14 \\ 0 & -3 \end{pmatrix}$

Therefore the square root of $\begin{pmatrix} 16 & -14 \\ 0 & 9 \end{pmatrix}$ gives four possible matrices

2. In this case, the determinant method doesn't apply here and it occurs when **a = -d** or **d = -a (it gives a diagonal matrix)**

When **a = - d**, "**g**" and "**f**" becomes zero

Proof:

Let **d = -a**

$$\begin{pmatrix} a & b \\ c & -a \end{pmatrix}\begin{pmatrix} a & b \\ c & -a \end{pmatrix} = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

$$\begin{pmatrix} a^2 + bc & ab - bd \\ ac - cd & a^2 + bc \end{pmatrix} = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

$$\begin{pmatrix} a^2 + bc & 0 \\ 0 & a^2 + bc \end{pmatrix} = \begin{pmatrix} e & f \\ g & h \end{pmatrix}$$

a^2 + bc = e ….. (i)

bc + a^2 = h ……. (ii) a, b, c, e, h, €Ẕ

a^2 = (e –bc) ….. (i)

Since, the two equations are the same, the system of equation gives an infinite number of solutions, thereby giving rise to many matrices assuming values for "**b**" and "**c**" such that **(e –bc)** gives a perfect square

10. NUMERICAL EXAMPLES

(1) Find the square root of $\begin{pmatrix} 33 & 0 \\ 0 & 33 \end{pmatrix}$

Solution

Since **g** = **f** = 0, then **a** = **-d**

Recall,

a^2 + bc = e

a^2 + bc = 33

It should be noted that **(e –bc)** should give a perfect square

Let **b** = 4, **c** = 2

a^2 + 4(2) = 33

a^2 = 33 - 8

a^2 = 25

a = ±5

If **a** = 5

Since **d** = **-a**

d = -5

:. The original matrix becomes $\begin{pmatrix} 5 & 4 \\ 2 & -5 \end{pmatrix}$ or $\begin{pmatrix} -5 & -4 \\ -2 & 5 \end{pmatrix}$

OR

International Journal of Mathematics and Physical Sciences Research ISSN 2348-5736 (Online)
Vol. 6, Issue 1, pp: (61-71), Month: April - September 2018, Available at: www.researchpublish.com

If **b** = 8, **c** = -2

$\mathbf{a}^2 + 8(-2) = 33$

$\mathbf{a}^2 = 33 + 16$

$\mathbf{a}^2 = 49$

$\mathbf{a} = \sqrt{49}$

$\mathbf{a} = \pm 7$

:. **d** = -7

:. The original matrix becomes $\begin{pmatrix} 7 & 8 \\ -2 & -7 \end{pmatrix}$ or $\begin{pmatrix} -7 & -8 \\ 2 & 7 \end{pmatrix}$

And so on…

(2) Find the square root of $\begin{pmatrix} 49 & 0 \\ 0 & 49 \end{pmatrix}$

Solution

Since **g = f = 0**, then **a = -d**

Recall,

$\mathbf{a}^2 + \mathbf{bc} = \mathbf{e}$

Let **b** = 0 and **c** = o

$\mathbf{a}^2 = 49$

a = ±7, **a** = 7

d = **-a** = -7

The original matrix becomes $\begin{pmatrix} 7 & 0 \\ 0 & -7 \end{pmatrix}$ or $\begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$ (since b = c = 0)

OR

If **b** = 3, **c** = -5

$\mathbf{a}^2 + 3(-5) = 49$

$\mathbf{a}^2 = 49 + 15$

$\mathbf{a}^2 = 64$

a = ±8

If **a** = 8, **d** = -8

:. The original matrix becomes $\begin{pmatrix} 8 & 3 \\ -5 & -8 \end{pmatrix}$ or $\begin{pmatrix} -8 & -3 \\ 5 & 8 \end{pmatrix}$

And so on…

In general, a matrix can have several square roots. The **uncertainty ratio** offers a fast and direct method for finding it. The example below shows the usefulness of the **uncertainty ratio**.

Find the square root of the matrix $\begin{pmatrix} 33 & 24 \\ 48 & 57 \end{pmatrix}$

Solution

Recall,

$$\frac{c}{b} = \frac{g}{f} = \frac{48}{24} \text{ or } \frac{24}{12} \text{ or } \frac{16}{8} \text{ or } \frac{8}{4} \text{ or } \frac{4}{2}$$

There are seven possible values for "**b**" and "**c**", so we test them using eqn **(i)** and **(iv)** and it is reduced to two possible values for "**b**" and "**c**",

c	**b**
8	4
4	2

There are two possible values for "**b**" and "**c**", so we solve using them

$\mathbf{a}^2 + \mathbf{bc} = \mathbf{e}$ ….. **(i)** $\mathbf{bc} + \mathbf{d}^2 = \mathbf{h}$ ……. **(iv)**

When **c** = 8, **b** = 4

$\mathbf{a}^2 + (8 \text{ x } 4) = 33$ $(8 \text{ x } 4) + \mathbf{d}^2 = 57$

$\mathbf{a}^2 + 32 = 33$ $32 + \mathbf{d}^2 = 57$

$\mathbf{a}^2 = 1$ $\mathbf{d}^2 = 25$

$\mathbf{a} = \pm 1$ $\mathbf{d} = \pm 5$

The original matrix would be of the form $\begin{pmatrix} \pm 1 & 4 \\ 8 & \pm 5 \end{pmatrix}$

The check the polarity of "**a**" and '**d**"

The determinant of the squared matrix Δ_2 is $\begin{vmatrix} 33 & 24 \\ 48 & 57 \end{vmatrix}$

$\Delta_2 = 33(57) – (48 \text{ x } 24)$

$\Delta_2 = 1881 - 1152$

$\Delta_2 = 729$

Recall,

$\Delta_1 = \pm\sqrt{\Delta_2}$

$\Delta_1 = \pm\sqrt{729}$

$\Delta_1 = \pm 27$

Where Δ_1 is the determinant of the original matrix.

The determinant of the original matrix Δ_1 is $\begin{vmatrix} \pm 1 & 4 \\ 8 & \pm 5 \end{vmatrix}$

$\Delta_1 = (\pm 1 \text{ x } \pm 5) – (8 \text{ x } 4)$

$\Delta_1 = \pm 5 – 32$

When the variable 5 is negative, it gives $-37 \neq \pm 27$

When the variable 5 is positive, it gives $-27 = -27$

So neither "**a**" or "**d**" takes minus (**principle 4**)

:. The original matrix becomes $\begin{pmatrix} 1 & 4 \\ 8 & 5 \end{pmatrix}$ or its negative $\begin{pmatrix} -1 & -4 \\ -8 & -5 \end{pmatrix}$

When, **c** = 4 and **b** = 2

$\mathbf{a}^2 + (4 \text{ x } 2) = 33$ $(4 \text{ x } 2) + \mathbf{d}^2 = 57$

$\mathbf{a}^2 + (8) = 33$ $(8) + \mathbf{d}^2 = 57$

$\mathbf{a}^2 = 33 - 8$ $\mathbf{d}^2 = 57 - 8$

$\mathbf{a}^2 = 25$ $\mathbf{d}^2 = 49$

$\mathbf{a}^2 = \pm\sqrt{25}$ $\mathbf{d}^2 = \pm\sqrt{49}$

$\mathbf{a} = \pm 5$ $\mathbf{d} = \pm 7$

The original matrix would be of the form $\begin{pmatrix} \pm 5 & 2 \\ 4 & \pm 7 \end{pmatrix}$

The check the polarity of "**a**" and '**d**"

The determinant of the squared matrix Δ_2 is $\begin{vmatrix} 33 & 24 \\ 48 & 57 \end{vmatrix}$

$\Delta_2 = 33(57) – (48 \text{ x } 24)$

$\Delta_2 = 1881 - 1152$

$\Delta_2 = 729$

Recall,

$\Delta_1 = \pm\sqrt{\Delta_2}$

$\Delta_1 = \pm\sqrt{729}$

$\Delta_1 = \pm 27$

Where Δ_1 is the determinant of the original matrix.

The determinant of the original matrix Δ_1 is $\begin{vmatrix} \pm 5 & 2 \\ 4 & \pm 7 \end{vmatrix}$

$\Delta_1 = (\pm 5 \text{ x } \pm 7) – (4 \text{ x } 2)$

$\Delta_1 = \pm 35 – 8$

When the variable 35 is negative, it gives -43 $\neq \pm 27$

When the variable 4 is positive, it gives 27 = 27

So neither "**a**" or "**d**" takes minus (**principle 4**)

:. The original matrix becomes $\begin{pmatrix} 5 & 2 \\ 4 & 7 \end{pmatrix}$ or it's negative $\begin{pmatrix} -5 & -2 \\ -4 & -7 \end{pmatrix}$

So the matrix gives four possible squared matrices.

11. CONCLUSION

The determinant method is mainly used to test the polarity of "**a**" and "**d**"

In **special case 1**, the value of "**(a + b)**" should be a factor of "**g**" since **a, b, c, e, g, h, €Z** and the matrix gives four possible square matrices.

In **special case 2**, "**(e –bc)**" or "**(h –bc)**" should give a perfect square and gives rise to many (infinite) solutions.

This approach to solving the square root of a 2x2 perfect square matrix is also applicable to a hermittian perfect square matrix of the same order.